

Effect of heat source on transient energy growth analysis of a thermoacoustic system



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ARTICLE INFO

Article history:

Received 8 June 2014

Accepted 28 September 2014

Keywords:

Thermoacoustic oscillation

Transient energy growth

Combustion instability

Premixed flame

Non-normality

Heat-to-sound conversion

ABSTRACT

Transient growth of disturbances in a non-normal combustion system could trigger thermoacoustic instability. In this work, the effect of heat source on triggering thermoacoustic instability in a system with Dirichlet or Neumann boundary conditions is investigated. For this a thermoacoustic model of a premixed laminar flame is developed. It is formulated in state-space by expanding flow disturbances via Galerkin series and coupling with a flame model, thus providing a platform on which to gain insight on the system stability behaviors. Transient energy growth analysis is then performed by discretizing the flame model into distributed monopole-like sound sources. It is shown that the thermoacoustic system is non-normal and associated with transient energy growth. Parametric studies are then conducted to study the effects of (1) the flame-acoustics interaction index $\mathcal{J}$, (2) the eigenmode number N , (3) the mean temperature ratio $\mathcal{T}$ between pre- and after-combustion regions on the system stability behavior and non-normality. It is found that the maximum transient energy growth $G_{\max}$ is increased with increased $\mathcal{J}$. Furthermore, the thermoacoustic system with more eigenmodes is found to be associated with a larger $G_{\max}$.

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1. Introduction

To achieve low NO_x emission and increased combustion efficiency, lean premixed pre-vaporized combustion technology is widely applied in aero-engine and gas turbines. However, they are more susceptible to thermoacoustic instability [1–4]. It occurs frequently in other combustion systems such as rocket motors, ramjets, boilers or furnaces. Such instability is generated due to the coupling between unsteady combustion process and acoustic disturbances present [5]. As an efficient monopole-like sound source, unsteady combustion produces acoustic waves [6]. And these pressure waves [4] propagate within the combustor and partially reflect from boundaries to arrive back at the combustion zone. When unsteady heat is added in phase with the pressure oscillations [7], acoustical energy increases until limit cycle oscillations [8] occur. Such oscillations are wanted in thermoacoustic engine systems [9–19]. However, they are undesirable in aero-engines and gas turbines, since the oscillations may be so intensive that they can cause structural damage, enhanced heat transfer and costly mission failure to the engine systems [5,20].

Thermoacoustic systems have been shown to be nonlinear [21,22] and non-normal [23–29]. The non-normality [26] is

characterized by non-orthogonal eigenmodes. It arises from unsteady heat release and/or the complex impedance boundary conditions. It has also been shown that in a linearly stable but non-normal thermoacoustic system, there can be significant transient energy growth of small perturbations before their eventual decay. If the disturbance transient growth is large enough, thermoacoustic instability might be triggered by causing such disturbances to grow to amplitudes high enough to make nonlinear effects significant. However, different energy measures are used in the literature to characterize transient growth [24,30–33].

Chu [31] argued that the disturbance energy in a viscous compressible flow should include the entropy fluctuations in addition to conventional acoustical energy, which consists of both pressure and velocity fluctuations. Morfey [32] suggested that the total disturbance energy should include the energy associated with fluctuations in vorticity and entropy, beside the classical acoustical energy. Nicoud and Poinot [34] argued that the classical Rayleigh criterion is incomplete in describing the significant sources of fluctuating energy in a combustion system with a flow. Giauque et al. [35] relaxed the definition of the disturbance energy from Myers by considering chemical species and heat release terms. Initially, the energy measure used to study the transient growth in thermoacoustic system included the acoustical energy only [24,25,30]. However, there is no consensus on the definition of the disturbance energy. In addition to acoustical energy, entropy

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fluctuations were introduced by Mariappan and Sujith [23] in the energy measure. Subramanian and Sujith [33] introduced the unsteady heat release fluctuations in the energy measure. However, the previous transient growth analysis was conducted by assuming there is no temperature difference between the pre- and after-combustion regions. Thus there is a need to investigate the mean temperature effect on quantifying the disturbance transient growth. Lack of such investigations partially motivated the present work.

Experimental investigations were performed to measure the transient growth rate of thermoacoustic systems. Mariappan et al. conducted such transient growth measurements on a Rijke tube [28], which is a simple but widely used platform [36–39] to study heat-to-sound conversion. Kim and Hochgreb [41] performed similar measurements on a lean-premixed gas turbine combustor in Cambridge. The transient energy growth of the disturbances due to the non-normality cannot be predicted by the classical linear stability theory [21], since it provides information only about the long-term evolution of the eigenmodes [1,27,29]. To eliminate the disturbance transient growth, classical linear controllers are not applicable [27,29,40].

In this work, a simplified thermoacoustic system with Dirichlet or Neumann boundary conditions [42] is considered. A premixed laminar flame is confined and leads to a mean temperature jump from pre- to after-combustion region. In Section 2, the system equations are developed in state-space. The flow perturbations are expanded by using Galerkin series and the flame front is tracked by using the classical G -equation [43,44]. And unsteady heat release from the flame is assumed to be caused by its surface variation, which results from the fluctuations of the oncoming flow velocity. Both nonlinear and linearized flame models are discussed. In Section 3, transient energy growth analysis of the thermoacoustic system is performed. The energy measure consists of both acoustical energy and the unsteady heat release fluctuations by discretizing the flame front into distributed monopole-like sound sources. In Section 4, the results are presented and the key findings are discussed.

2. Description of the thermoacoustic model

In the present work, we consider a simplified combustion system with a monopole-like flame and Dirichlet boundary conditions implemented as shown in Fig. 1.

A premixed laminar conical-shaped flame acts as an acoustically compact heat source. It is modeled as a thin sheet and described by using Dirac delta function in mathematics. The flow variables with a tilde are the instantaneous quantities, which consist of a mean part denoted with an overbar and a fluctuating

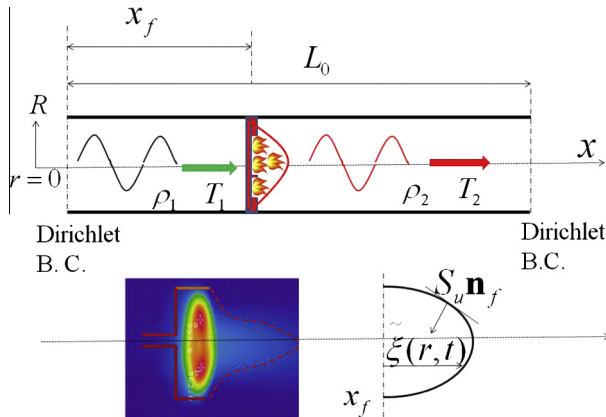


Fig. 1. Schematic of the combustion system with an acoustically compact premixed flame confined and Dirichlet boundary conditions applied on both inlet and outlet.

part. The dimensional governing equations of the thermoacoustic systems comprise:

$$\bar{\rho} \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0 \quad (1)$$

$$\frac{1}{\bar{c}^2} \frac{\partial^2 p}{\partial t^2} + \frac{\zeta}{\bar{c}^2} \frac{\partial p}{\partial t} - \frac{\partial^2 p}{\partial x^2} = \frac{(\gamma - 1)}{\bar{c}^2} \frac{\partial}{\partial t} (Q_f(t) \delta(x - x_f)) \quad (2)$$

where $\bar{c}$ is the sound speed, L_0 is the length of the duct, γ is the ratio of specific heats, $\delta(x)$ is dirac delta function describing the localized flame or the actuator, Q_f denotes the unsteady heat release from the flame.

Expanding the pressure perturbation as Galerkin series [25,27,30] gives,

$$p(x, t) = \sum_{n=1}^N \frac{-1}{\kappa_n \omega_n} \dot{\eta}_n(t) \psi_n(x) \quad (3)$$

where

$$\kappa_n = \left[\int_0^{x_f} \psi_n^2(x) dx + \int_{x_f}^{L_0} \psi_n^2(x) dx \right]^{1/2} \quad (4)$$

N denotes the eigenmode number. Overdot denotes the time derivative. The function $\psi_n(x)$ are the eigen-solutions of the homogeneous wave equation,

$$\frac{1}{\bar{c}^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0 \quad (5)$$

Since the presence of the flame results in the mean temperature undergoing a jump from the region $0 \leq x < x_f$ to $x_f < x \leq L_0$, substituting Eq. (3) into Eq. (5) gives

$$\frac{\omega_n^2}{\bar{c}_1^2} \psi_n + \frac{d^2 \psi_n}{dx^2} = 0, \quad 0 \leq x < x_f \quad \text{and} \quad \frac{\omega_n^2}{\bar{c}_2^2} \psi_n + \frac{d^2 \psi_n}{dx^2} = 0, \quad x_f < x \leq L_0 \quad (6)$$

Applying the pressure continuity and velocity jump across the flame and the Dirichlet boundary conditions at $x = 0$ and $x = L_0$ [25,27,30], i.e. $p(x, t)|_{x=0} = 0$ and $p(x, t)|_{x=L_0} = 0$ leads to

$$\psi_n(x) = \begin{cases} \sin\left(\frac{\omega_n x}{\bar{c}_1}\right) & 0 \leq x < x_f \\ -\sin\left(\frac{\omega_n(x-L_0)}{\bar{c}_2}\right) \frac{\sin(\omega_n x_f / \bar{c}_1)}{\sin(\omega_n(L_0-x_f)/\bar{c}_2)} & x_f < x \leq L_0 \end{cases} \quad (7)$$

that satisfies the condition of orthogonality, i.e. $\int_0^{L_0} \psi_m(x) \psi_n(x) dx = 0$ for $m \neq n$. Further analysis illustrates that ω_n is related to x_f in the form of

$$\begin{aligned} \bar{c}_2 \sin\left(\frac{\omega_n x_f}{\bar{c}_1}\right) \cos\left(\frac{\omega_n(L_0-x_f)}{\bar{c}_2}\right) \\ = -\bar{c}_1 \sin\left(\frac{\omega_n(L_0-x_f)}{\bar{c}_2}\right) \cos\left(\frac{\omega_n x_f}{\bar{c}_1}\right) \end{aligned} \quad (8)$$

Eq. (8) can be used to characterize the frequency ω_n of the eigenmode (see Fig. 2).

Substituting Eq. (7) into Eq. (3) yields

$$p(x, t) = \begin{cases} -\sum_{n=1}^N \frac{\dot{\eta}_n(t)}{\kappa_n \omega_n} \sin\left(\frac{\omega_n x}{\bar{c}_1}\right) & 0 \leq x < x_f \\ \sum_{n=1}^N \frac{\dot{\eta}_n(t)}{\kappa_n \omega_n} \sin\left(\frac{\omega_n(x-L_0)}{\bar{c}_2}\right) \frac{\sin(\omega_n x_f / \bar{c}_1)}{\sin(\omega_n(L_0-x_f)/\bar{c}_2)} & x_f < x \leq L_0 \end{cases} \quad (9)$$

Replacing $p(x, t)$ in the momentum Eq. (1) with Eq. (9) gives

$$u(x, t) = \begin{cases} \sum_{n=1}^N \frac{\dot{\eta}_n(t)}{\rho_1 \bar{c}_1 \kappa_n} \cos\left(\frac{\omega_n x}{\bar{c}_1}\right) & 0 \leq x < x_f \\ -\sum_{n=1}^N \frac{\dot{\eta}_n(t)}{\rho_2 \bar{c}_2 \kappa_n} \cos\left(\frac{\omega_n(L_0-x)}{\bar{c}_2}\right) \frac{\sin(\omega_n x_f / \bar{c}_1)}{\sin(\omega_n(L_0-x_f)/\bar{c}_2)} & x_f < x \leq L_0 \end{cases} \quad (10)$$

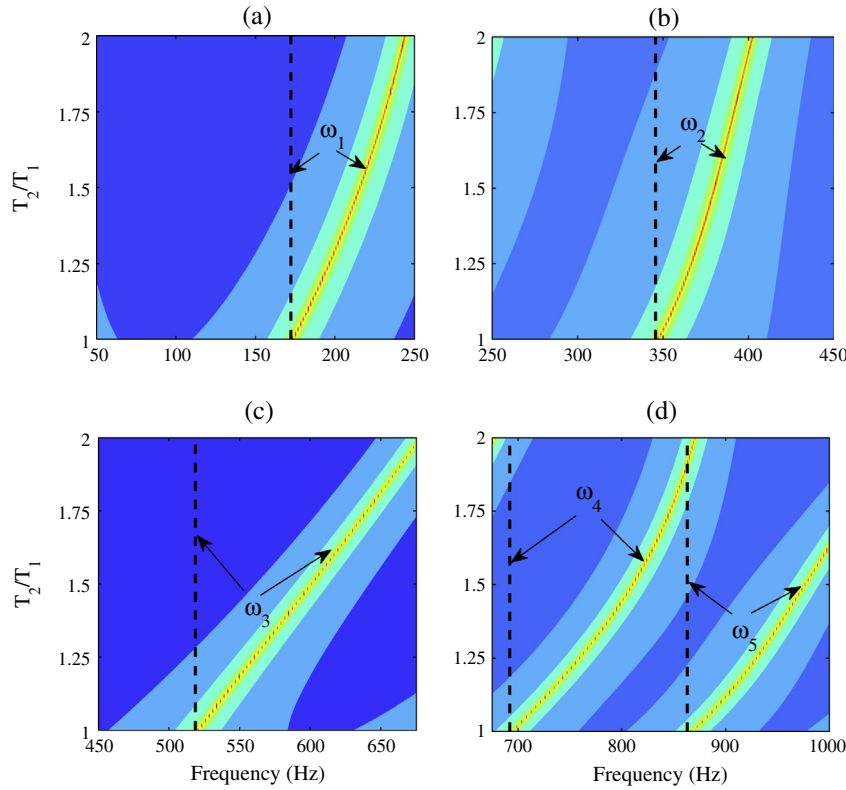


Fig. 2. Variation of predicted eigenmode frequencies with temperature ratio $\bar{T}_2/\bar{T}_1$ in a thermoacoustic system with Dirichlet or Neumann boundary conditions.

Now substituting the Galerkin series expansion form of the pressure perturbation, i.e. Eq. (3) into Eq. (2) leads to

$$\sum_{n=1}^N \frac{-1}{\kappa_n \omega_n} \left(\frac{d^2 \dot{\eta}_n}{dt^2} + \zeta \frac{d \dot{\eta}_n}{dt} + \omega_n^2 \dot{\eta}_n \right) \psi_n(x) = (\gamma - 1) \frac{\partial}{\partial t} [Q_f(t) \delta(x - x_f)] \quad (11)$$

Multiplying Eq. (11) with $\psi_n(x)$ and integrating from $x = 0$ to $x = L_0$ gives

$$\frac{d^2 \dot{\eta}_n}{dt^2} + \zeta \frac{d \dot{\eta}_n}{dt} + \omega_n^2 \dot{\eta}_n = \frac{(1 - \gamma) \omega_n}{\kappa_n} \frac{\partial}{\partial t} \left[\int_0^{L_0} Q_f(t) \delta(x - x_f) \psi_n(x) dx \right] \quad (12)$$

A little algebra shows that

$$\frac{d^2 \eta_n}{dt^2} + \zeta \frac{d \eta_n}{dt} + \omega_n^2 \eta_n = \frac{\omega_n (1 - \gamma)}{\kappa_n} \left[Q_f(t) \sin \left(\frac{\omega_n x_f}{\bar{c}_1} \right) \right] \quad (13)$$

where $\kappa_n = \left[\int_0^{L_0} \psi_n^2(x) dx \right]^{1/2}$. When $T_1 = T_2$ (i.e. no mean temperature gradient), $\bar{c}_1 = \bar{c}_2$ and $\kappa_n = (L/2)^{1/2}$ is a constant [25]. However, when $T_1 \neq T_2$, κ_n depends on the mode frequency ω_n and the flame position x_f . The damping ζ is taken into account by assigning a damping parameter ζ_n to each mode as

$$\zeta_n = \frac{1}{2\pi} \left[\mathcal{C}_1 \frac{\omega_n}{\omega_1} + \mathcal{C}_2 \sqrt{\frac{\omega_1}{\omega_n}} \right] \quad (14)$$

The instantaneous heat release rate $\tilde{Q}_f(t)$ from the conical-shaped flame [43] is given as

$$\tilde{Q}_f(t) = \bar{Q}_f + Q_f(t) = \rho_n S_u \tilde{A}_f \Delta H_r = \rho_n S_u (\bar{A}_f + A_f) \Delta H_r \quad (15)$$

where ρ_n is the mixture density, S_u is the flame speed, $\tilde{A}_f$ is the instantaneous flame surface area and ΔH_r is the heat of reaction per unit mass of the mixture. The heat release rate expressed in the sum of a mean and perturbation can be shown as

$$\frac{Q_f(t)}{\bar{Q}_f} = \frac{A_f(t)}{\bar{A}_f} \quad (16)$$

To determine the flame surface area, the flame dynamics are captured using the classical G-equation [43,44]. The oncoming flow to the flame is assumed to be axisymmetric and combustion is assumed to occur on a thin surface whose axial position at radius r is given by $x = \tilde{\xi}(r, t)$ as shown in Fig. 1. The flame initiation surface is then described by $G(x, r, t) = 0$, where $G(x, r, t) = x - \tilde{\xi}(r, t)$. Assuming the surface propagates itself in the direction of its normal $\mathbf{n}$ i.e. $\nabla G/|\nabla G|$ into the unburnt fluid at a constant speed S_u gives $\partial G/\partial t = -\tilde{\mathbf{u}} \cdot \nabla G + S_u |\nabla G|$ [43,44]. With the assumption that the density change across the surface $G(x, r, t) = 0$ is negligible, the particle velocity is given by that in the oncoming flow $(\tilde{u}_f, 0)$ and the equation for $\tilde{u}_f$, the velocity distribution at the flame simplifies to

$$\tilde{u}_f = \frac{\partial \tilde{\xi}(r, t)}{\partial t} + S_u \sqrt{1 + \left(\frac{\partial \tilde{\xi}(r, t)}{\partial r} \right)^2} \quad (17)$$

If $\tilde{\xi}(r, t)$ is known, the instantaneous flame surface area $\tilde{A}_f(t)$ can be readily calculated by

$$\tilde{A}_f(t) = \bar{A}_f + A_f(t) = \int_0^{2\pi} d\theta \int_0^{R_0} r \sqrt{1 + \left(\frac{\partial(\tilde{\xi} + \xi(r, t))}{\partial r} \right)^2} dr \quad (18)$$

where R_0 are the radius of the duct. It can then be shown that the mean and fluctuating parts of the flame surface area are given as

$$\bar{A}_f = \int_0^{R_0} 2\pi r \sqrt{1 + \left(\frac{d\tilde{\xi}(r)}{dr} \right)^2} dr = \frac{\tilde{u}_f}{S_u} \pi R_0^2 \quad (19)$$

$$\begin{aligned} A_f(t) &\approx \int_{R_a}^{R_b} 2\pi r \frac{\frac{\partial \xi'(r, t)}{\partial r} \frac{\partial \tilde{\xi}(r)}{\partial r}}{\sqrt{1 + \left(\frac{\partial \tilde{\xi}(r)}{\partial r} \right)^2}} dr = -2\pi \sqrt{1 - \frac{S_u^2}{\tilde{u}_f^2}} \int_0^{R_0} \xi dr \\ &= \chi \sum_{k=1}^K \lambda_k \xi(r, t) \delta r \end{aligned} \quad (20)$$

where $\chi = -2\pi \sqrt{1 - S_u^2/\tilde{u}_f^2}$. The flame front is discretized into K flame elements. Each behaves like a monopole sound source. λ_k is

the weight factor corresponding to the trapezoidal integration formula as given as $\lambda_k = 1/2$, as $k = 1$ or K , while $k \neq 1$ or K , $\lambda_k = 1$. After discretization, it can be rewritten as

$$\frac{d\zeta_k}{dt} = \sum_{n=1}^N \frac{\eta_n}{\bar{\rho}_1 \bar{c}_1 \kappa_n} \cos\left(\frac{\omega_n x_f}{\bar{c}_1}\right) - S_u \frac{\zeta_k - \zeta_{k-1}}{\delta r} \quad (21)$$

The boundary and initial conditions are

$$\tilde{\zeta}(K\delta r, t) = 0, \quad \tilde{\zeta}(r, t = 0) = \tilde{\zeta}(r) \quad (22)$$

It is worth noting that the analytical method present in this work can also be applied to a combustion system with Neumann boundary conditions as discussed in Appendix A.

3. Transient energy growth analysis of the thermoacoustic system

3.1. Linearized thermoacoustic system

The governing Eqs. (13) and (21) of the thermoacoustic system can be formulated in state-space form by linearizing the flame model via discretizing into K flame elements as

$$\frac{d\mathbf{x}}{dt} = \frac{d}{dt} \begin{pmatrix} \mathbf{x}_A \\ \mathbf{x}_F \end{pmatrix} = \begin{pmatrix} \mathbf{L}^{TA} & \mathbf{L}^{TF} \\ \mathbf{L}^{BA} & \mathbf{L}^{BF} \end{pmatrix} \begin{pmatrix} \mathbf{x}_A \\ \mathbf{x}_F \end{pmatrix} = \mathbf{L}\mathbf{x} \quad (23)$$

where the coefficient matrices $\mathbf{L}^{TA}$, $\mathbf{L}^{TF}$, $\mathbf{L}^{BA}$ and $\mathbf{L}^{BF}$ are given in the Appendix B. $\mathbf{L}$ is a $(2N + K) \times (2N + K)$ matrix. $\mathbf{x}$ is the state vector as given as

$$\mathbf{x} = \begin{pmatrix} \underbrace{\kappa_1 \eta_1, \frac{\kappa_1 \dot{\eta}_1}{\omega_1}, \kappa_2 \eta_2, \frac{\kappa_2 \dot{\eta}_2}{\omega_2}, \dots, \kappa_N \eta_N, \frac{\kappa_N \dot{\eta}_N}{\omega_N}}_{\mathbf{x}_A}, \underbrace{\lambda_1 \zeta_1 \Delta r, \lambda_2 \zeta_2 \Delta r, \dots, \lambda_K \zeta_K \Delta r}_{\mathbf{x}_F} \end{pmatrix} \quad (24)$$

The linearized thermoacoustic system can be shown to be non-normal due to the fact that $\mathbf{L}_{(2N+K) \times (2N+K)}$ does not commute with its adjoint [25], i.e. $\mathbf{L}\mathbf{L}^* \neq \mathbf{L}^*\mathbf{L}$. The system can also be shown to be non-linear if the nonlinear flame model is used.

Note that the linearized thermoacoustic system is formulated in state space. This enables the energy of the flow disturbances to be defined clearly as discussed later and the transient growth to be calculated easily. In addition, the linearized governing equations can be used to determine the stability behaviors of the thermoacoustic eigenmodes. This will be helpful on the design of a stable combustion system, of which the eigenmodes stability need to be predicted.

3.2. Definition of a measure of total disturbance energy

To characterize the transient growth of disturbances in the system, it is necessary to define a measure. The thermoacoustic system is involved with unsteady heat release Q_f , which is a monopole-like sound source and related to the entropy fluctuation [31]. As discussed in the previous works [1,31,33], the unsteady heat release can be considered as part of the total energy contained in the disturbances present in the thermoacoustic system. Thus the energy due to the monopole-like flame can be shown as

$$E_f(t) = \int_0^{L_0} (Q_f(t) \delta(x - x_f))^2 dx = \left(\chi \frac{\bar{Q}_f}{\bar{A}_f} \right)^2 \sum_{k=1}^K (\lambda_k \zeta_k \delta r)^2 \quad (25)$$

$$= \mathcal{J} \sum_{k=1}^K (\lambda_k \zeta_k \delta r)^2 \quad (26)$$

where $\mathcal{J} \geq 0$ is an index denoting the flame-acoustics interaction intensity. Its effect on the system stability behavior and transient growth is studied in the following section.

Besides the unsteady heat release fluctuation, the system is also associated with acoustic waves propagating along the combustor. The acoustical energy [25,26,29] $E_a(x_f, t)$ per unit cross-sectional area consists of N eigenmodes. It can also be separated into a kinetic component E_k , and a potential component E_p . The definition of $E_{\text{tot}}(x_f, t)$ is given as

$$\begin{aligned} E_a(x_f, t) &= \int_0^{x_f} \left(\frac{p_1^2(x, t)}{2\bar{\rho}_1 \bar{c}_1^2} + \frac{\bar{\rho}_1 u_1^2(x, t)}{2} \right) dx \\ &\quad + \int_{x_f}^L \left(\frac{p_2^2(x, t)}{2\bar{\rho}_2 \bar{c}_2^2} + \frac{\bar{\rho}_2 u_2^2(x, t)}{2} \right) dx \\ &= \frac{1}{2\gamma \bar{p}} \sum_{n=1}^N \left(\eta_n^2 + \frac{\dot{\eta}_n^2}{\omega_n^2} \right) = \sum_{n=1}^N E_n(x_f, t) \end{aligned} \quad (27)$$

It is apparent that $E_a(x_f, t)$ is a function of both flame position x_f and time t . Note that the state equation of perfect gas $\gamma \bar{p} = \bar{\rho}_1 \bar{c}_1^2 = \bar{\rho}_2 \bar{c}_2^2$ is used to derive Eq. (27).

The total energy E_{tot} due to the unsteady heat release and acoustic fluctuations includes the contributions from E_a and E_f . Thus E_{tot} can be expressed as

$$E_{\text{tot}}(t) = E_a(t) + E_f(t) = \frac{1}{2\gamma \bar{p}} \sum_{n=1}^N \left(\eta_n^2 + \frac{\dot{\eta}_n^2}{\omega_n^2} \right) + \mathcal{J} \sum_{k=1}^K (\lambda_k \zeta_k \delta r)^2 \quad (28)$$

According to Eq. (23), $\mathbf{L}$ is the linear stability operator of the linearized system. The solution to Eq. (23) is given as

$$\mathbf{x} = \exp(\mathbf{L}t)\mathbf{x}(0) \quad (29)$$

The square of the norm $\|\mathbf{x}\|$ is given as

$$\|\mathbf{x}\|^2 = \sum_{n=1}^N \kappa_n^2 \left[\eta_n^2 + \frac{\dot{\eta}_n^2}{\omega_n^2} \right] + \sum_{k=1}^K [\lambda_k \zeta_k \delta r]^2 = \|\exp(\mathbf{L}t)\mathbf{x}(0)\|^2 \quad (30)$$

Comparing the definition of the total energy as given in Eq. (28) with Eq. (30) reveals that $E_{\text{tot}}(x_f, t)$ is related to $\|\mathbf{x}\|$ as

$$E_{\text{tot}}(x_f, t) = \|\mathbf{Q}\mathbf{x}(t)\|^2 \quad (31)$$

where $\mathbf{Q}$ is a $2N \times 2N$ diagonal matrix as given in Appendix C.

For a given x_f , the maximum total energy growth rate at time t , optimized over all possible initial perturbations can be expressed as

$$G(x_f, t) = \max_{E_{\text{tot}}(x_f, 0)} \frac{E_{\text{tot}}(x_f, t)}{E_{\text{tot}}(x_f, 0)} = \frac{\|\mathbf{Q}\mathbf{x}(t)\|^2}{\|\mathbf{Q}\mathbf{x}(0)\|^2} = \|\exp(\mathbf{Q}\mathbf{L}\mathbf{Q}^{-1}t)\|^2 \quad (32)$$

Thus the global maximum energy growth factor over a given time interval $[0, t]$ is

$$G_{\text{max}}(x_f) = \max_{t \rightarrow \infty} \{G(x_f, t)\} \quad (33)$$

$\|\exp(\mathbf{Q}\mathbf{L}\mathbf{Q}^{-1}t)\|$ is the 2-norm of the matrix exponential $\exp(\mathbf{Q}\mathbf{L}\mathbf{Q}^{-1}t)$. To calculate $G_{\text{max}}(x_f, t)$, singular values decomposition method (SVD) is used to determine the singular values of $\exp(\mathbf{Q}\mathbf{L}\mathbf{Q}^{-1}t)$. Detailed information about SVD can be found in the previous works [26,30].

When $G_{\text{max}} > 1$, then the disturbance energy is increased for some time t (known as transient growth). However, if $G(t) \rightarrow +\infty$ as time tends to infinity, then the thermoacoustic system is linearly unstable. This means that the real part of the eigenvalues of $\mathbf{Q}\mathbf{L}\mathbf{Q}^{-1}$ are positive. When $G_{\text{max}} = 1$, there is no transient growth of the disturbance energy (also known as strict dissipativity). This ensures that the rate of total disturbance energy change at any instant of time is non-positive, i.e.,

$$\frac{dE_{\text{tot}}}{dt} = \frac{1}{\gamma \bar{p}} \sum_{n=1}^N \left[\eta_n \frac{d\eta_n}{dt} + \left(\frac{\dot{\eta}_n}{\omega_n} \right) \frac{d}{dt} \left(\frac{\dot{\eta}_n}{\omega_n} \right) \right] + \mathcal{J} \sum_{k=1}^K \frac{d}{dt} [(\lambda_k \zeta_k \delta r)^2] \leq 0 \quad (34)$$

4. Results and discussion

4.1. Nonlinear response of the thermoacoustic system

The variation of the linearized flame surface area with disturbances frequency ω is shown in Fig. 3. It can be seen that the

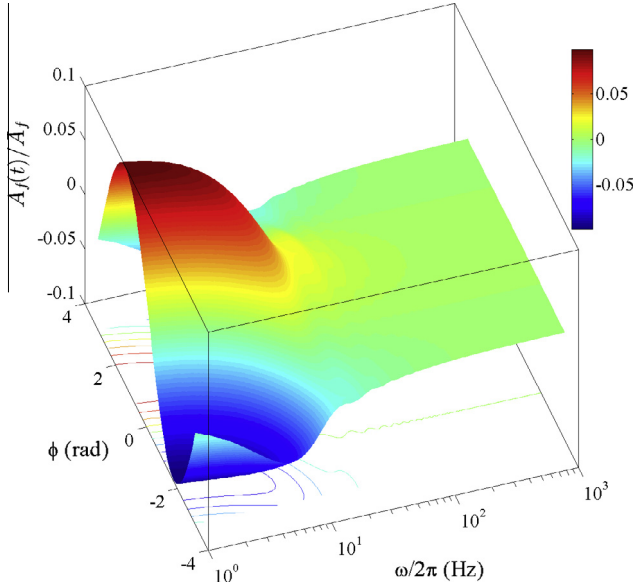


Fig. 3. Flame surface area variation $A_f(t)/\bar{A}_f$ with ω and ϕ , during one period of the acoustic disturbance $-\pi \leq \phi \leq \pi$, as the oncoming flow disturbance is given as $u(t) = \bar{u}_c(1 + 0.2 \sin(\omega t))$.

amplitude of flame surface area ratio $A_f(t)/\bar{A}_f$ is reduced with increased ω and periodically varied with ϕ . This means that flow disturbances at higher frequency pass through the flame smoothly. However, the flame responds strongly to the flow disturbances at lower frequency.

When the nonlinear flame model is combined with the acoustic model, time evolution of the thermoacoustic oscillations is determined, as shown in Fig. 4(a). It can be seen that the initial disturbances quickly grow into a nonlinear limit cycle. The frequency spectrum is shown in Fig. 4(b). It is apparent that there are harmonics present, indicating the system nonlinearity. Fig. 4(c) illustrates the mode-shapes at ω_1 and ω_2 in terms of velocity fluctuations. Velocity antinodes are located at both ends, as expected.

In order to gain insight on the generation of the limit cycle oscillations, Rayleigh criterion or index is generally used to characterize the heat-to-sound energy conversion process. For this, we define the phase difference θ_{pQ} between pressure fluctuation and unsteady heat release rate as Rayleigh index as given as

$$\theta(\omega, t) = \arccos \left(\frac{\int_0^t \frac{p(\omega_1, \Theta) Q(\omega_1, \Theta) d\Theta}{\sqrt{\int_0^{2\pi/\omega_1} p^2(\omega_1, \Theta) d\Theta} \sqrt{\int_0^{2\pi/\omega_1} Q^2(\omega_1, \Theta) d\Theta}} d\Theta \right) \quad (35)$$

The variation of the phase θ_{pQ} is shown in Fig. 4(d). It can be seen that initially the phase difference is close to 90 degree, i.e. pressure fluctuations and unsteady heat release are out of phase. Thus the initial disturbances decay (see Fig. 4(a) at $t < 0.3$ s). However, with the phase difference reduced, i.e. the pressure fluctuations and unsteady heat release in phase, the acoustical energy is increased

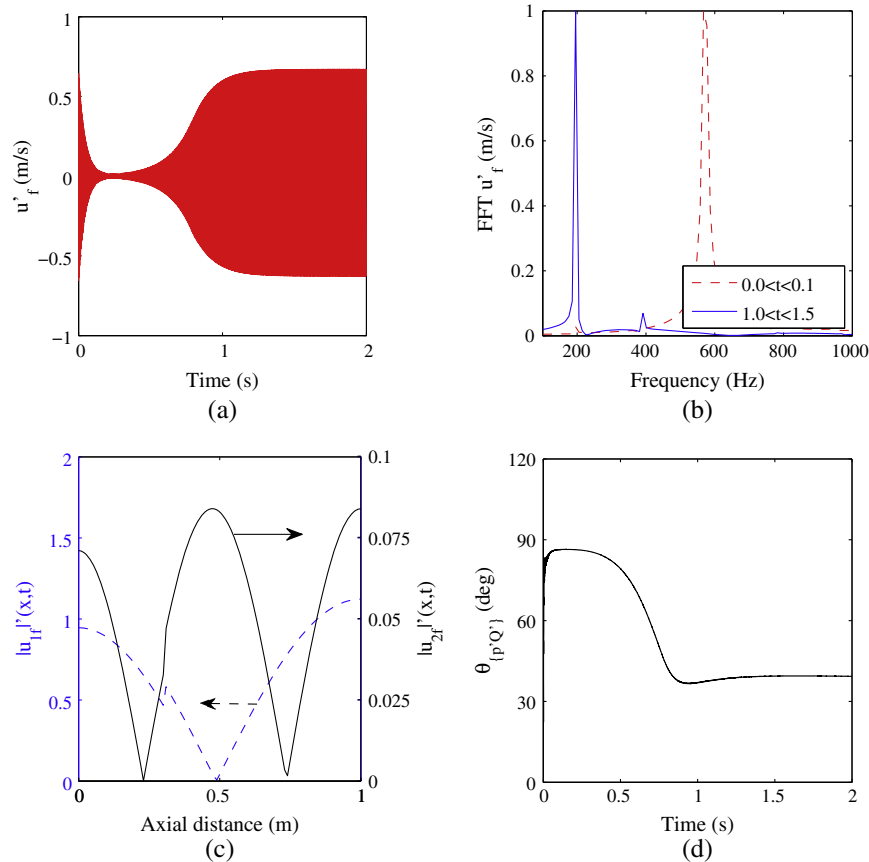


Fig. 4. Self-sustained combustion oscillations (a) velocity limit cycle, (b) frequency spectrum, (c) velocity mode shapes at ω_1 and ω_2 , and (d) the phase difference θ between unsteady heat release $Q_f(t)$ and pressure fluctuation $p(x_f, t)$ at ω_1 with time.

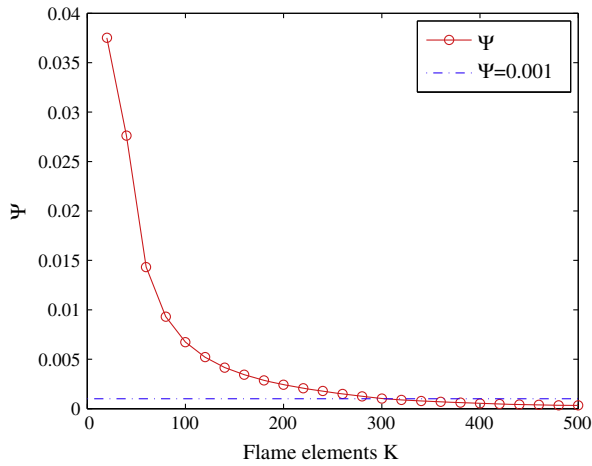


Fig. 5. Variation of Ψ with the flame elements number K in a thermoacoustic system, as $x_f = 0.1$, $L_0 = 1.0$, $T_1 = 300$ K, $\mathcal{J} = 1.1 \times 10^7$ and $N = 6$.

and finally 'saturated'. And limit cycle thermoacoustic oscillations are produced. This is consistent with the well-known Rayleigh Criterion.

4.2. Parametric study of transient energy growth

Before we conduct transient energy growth analysis, it is important to define a measure to check the convergence with increased number of flame elements. The number of flame elements is varied in steps of 20 and the following relative change is calculated

$$\psi = \frac{|G_{\max}(K) - G_{\max}(K - 20)|}{G_{\max}(K)} \quad (36)$$

Fig. 5 shows the variation of Ψ with a change in the number of flame elements K . It can be seen that the relative change is less than 0.1% for $K = 350$. Thus the flame elements number is chosen to be 400 in our following analysis.

Parametric studies of transient energy growth are then performed by using the thermoacoustic model described above. There

are 3 key parameters: (1) the flame-acoustics interaction index $\mathcal{J}$, (2) the eigenmode number N , and (3) the mean temperature ratio $\mathcal{T}$. The effects of such parameters on the transient energy growth are investigated by considering one parameter at a time, while others remain unchanged.

Let's first consider the effect of the flame-acoustics interaction index $\mathcal{J}$ on the system non-normality and $G_{\max}$. Fig. 6 shows the variation of the transient growth $G(t)$ in two thermoacoustic systems, as the flame-acoustics interaction index $\mathcal{J}$ is set to 3 different values. One is associated with no mean temperature ratio $\mathcal{T} = 1$. The other is with $\mathcal{T} = 2$. Note that The flame-acoustics interaction index is randomly chosen for illustration.

It can be seen from both systems that when the interaction index $\mathcal{J}$ is large, the thermoacoustic system is linearly unstable with the maximum transient growth $G_{\max} \rightarrow +\infty$, as denoted by the solid line. This means that the eigenmodes are associated with real positive parts. As the flame-acoustics interaction index $\mathcal{J}$ is decreased, the disturbances present in the system undergo transient growth, i.e. $G_{\max} > 1$, as denoted by the dash line in the figure. However, further decreasing $\mathcal{J}$ leads to $G_{\max} = 1.0$. This indicates the system is associated with decaying orthogonal eigenmodes. Similar behaviors are observed in the system with $\mathcal{T} = 2$. However, the gradient of energy growth $G(t)$ is different. Detailed study on the mean temperature effect is shown in the following figures.

The eigenmode number N is then considered. Fig. 7 shows the variation of the transient growth $G(t)$ in two thermoacoustic systems. One is involved with one eigenmode, i.e. $N = 1$. The other is with 6 eigenmodes, i.e. $N = 6$. It can be seen that with the number N of the eigenmodes increased, the maximum transient growth $G_{\max}$ is slightly increased. This is most likely due to the increased degrees of freedom present in the thermoacoustic systems. The increase of $G_{\max}$ indicates the intensified energy exchange between various eigenmodes. And the non-normality of the thermoacoustic system is increased. Fig. 7(b) shows the gradient of transient energy growth in the thermoacoustic system with 6 eigenmodes is larger than that with one eigenmode. This reveals that the energy conversion is intensified in the thermoacoustic systems with more eigenmodes present. This is most likely due to the increased degrees of freedom.

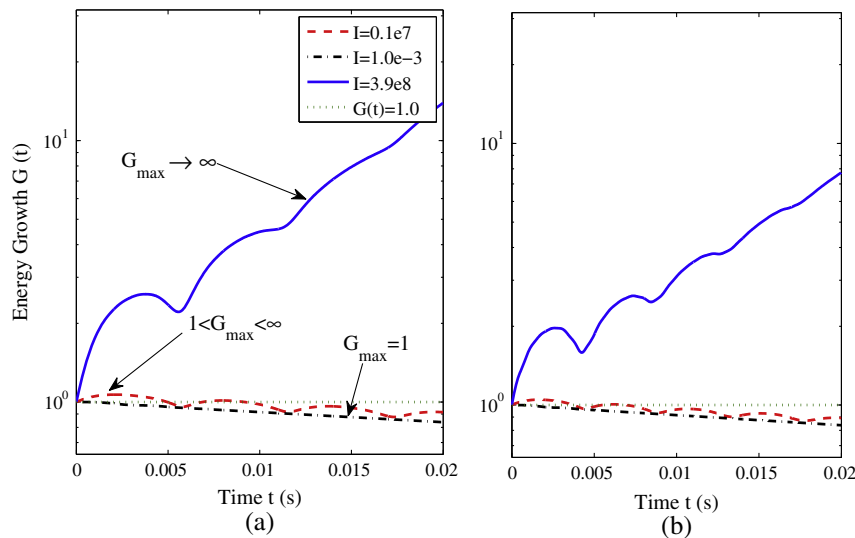


Fig. 6. Time evolution of transient growth $G(t)$ in thermoacoustic systems with ($\mathcal{T} > 1$) and without ($\mathcal{T} = 1$) mean temperature gradient, as the flame-acoustics interaction index $\mathcal{J}$ is set to 3 different values: (a) $\mathcal{J} = 1$, $x_f = 0.1$, $T_1 = 300$, $L_0 = 1.0$ m, $K = 400$ and $N = 6$, and (b) $\mathcal{J} = 2$, $x_f = 0.1$, $T_1 = 300$, $L_0 = 1.0$ m, $K = 400$ and $N = 6$.

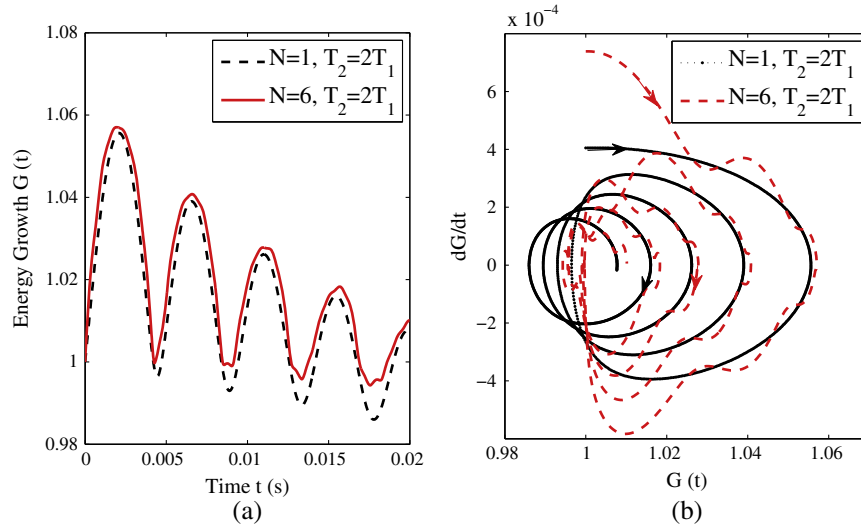


Fig. 7. (a) Time evolution of the transient energy growth $G(t)$, and (b) variation of the transient growth gradient $\dot{G}(t)$ with $G(t)$, as $x_f = 0.1$, $T_1 = 300$ K, $K = 400$, $L_0 = 1.0$ m, and $\mathcal{F} = 1.1 \times 10^7$.

Further analysis is conducted to gain insight on the effect of mean temperature ratio $\mathcal{F}$. Two thermoacoustic systems are considered. One is associated with 1 eigenmode and the other is with 6 eigenmodes. Fig. 8(a) and (b) show the variation of $G(t)$ with time, when $N = 1$ and $N = 6$ respectively. It can be seen that the maximum transient growth $G_{\max}$ at $\mathcal{F} = 2$ is smaller than that at $\mathcal{F} = 1$. This means that for a given $\mathcal{F}$, the maximum transient growth $G_{\max}$ is increased with decreased mean temperature ratio $\mathcal{F}$. And the thermoacoustic system is more susceptible to trigger thermoacoustic instability.

From an engineering point of view, it is important to know the ‘safest’ heat source position, where the heat source is less susceptible to trigger combustion instability. To determine such positions, the transient growth analysis is applied by calculating $G_{\max}$ via varying x_f . This is due to the fact that the maximum transient growth $G_{\max}$ is shown to be nonlinearly dependent on the heat

source position x_f/L_0 . The variation of $G_{\max}$ with x_f is shown in Fig. 9 at two different frequencies. It can be seen that placing the heat source at velocity node for example $x_f = 0.5$ leads to $G_{\max}$ to be minimized. In other words, the ‘safest’ heat source location corresponding to ω_1 is at the middle of the combustor. This finding is consistent with the linearized modal analysis of the thermoacoustic system [21,38]. It confirms that the model can be used to study the stability of the thermoacoustic system and to characterize the transient energy growth.

In summary, the thermoacoustic system is shown to be non-normal and associated with disturbances transient energy growth, when the flame and acoustics fluctuations are interacted. And thermoacoustic instability might be triggered by the disturbances transient growth. With the flame-acoustics interaction index increased, the system’s stability behavior is changed from linearly stable but with transient growth to linearly unstable.

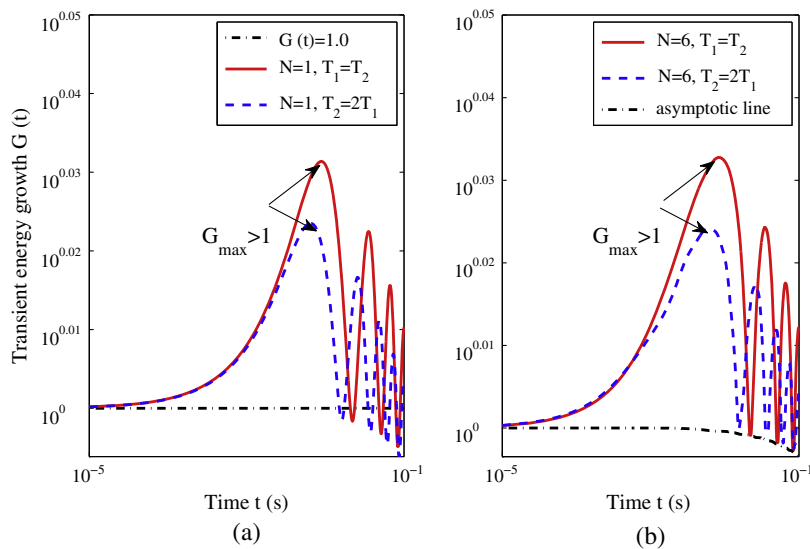


Fig. 8. Variation of transient energy growth $G(t)$ with time in the thermoacoustic system: (a) with one eigenmode, and (b) with six eigenmodes, as $x_f = 0.1$, as $L_0 = 1$ m, $T_1 = 300$ K, and $K = 400$ and $\mathcal{F} = 1.0 \times 10^6$.

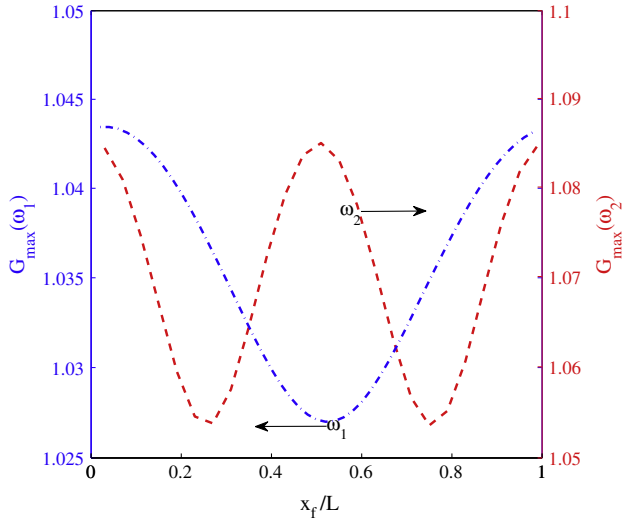


Fig. 9. Variation of maximum transient energy growth $G_{\max}$ with heat source location x_f in the thermoacoustic system: (a) ω_1 , and (b) ω_2 , as $L_0 = 1$ m, $T_1 = 300$ K, and $K = 400$ and $\mathcal{S} = 1.0 \times 10^6$.

5. Conclusions

The non-normal interaction between a premixed laminar flame and acoustic waves is studied in this work. The non-normality is characterized by transient energy growth of disturbances present in the thermoacoustic system. Unlike previous works, the unsteady heat release fluctuation is considered as part of the disturbance present by discretizing the flame front into distributed monopole-like source. The system equations are formulated in state-space by expanding flow disturbances via Galerkin series and discretizing flame front for transient energy growth analysis. Parametric studies are then conducted to study the effects of (1) the flame-acoustics interaction index $\mathcal{S}$, (2) the eigenmode number N , and (3) the mean temperature ratio $\mathcal{T}$ between pre- and after-combustion regions on the system non-normality. It is shown that with increased eigenmode number N , the maximum transient growth $G_{\max}$ is slightly increased. This is most likely due to the increased degrees of freedom present in the system. In addition, the mean temperature ratio $\mathcal{T}$ is found to play an important role in determining the maximum transient growth $G_{\max}$ and the system stability behavior. $G_{\max}$ is shown to decrease with increased $\mathcal{T}$, as flame-acoustics interaction index $\mathcal{S}$ is small. However, as $\mathcal{S}$ is greater than a 'critical' value, the thermoacoustic system becomes linearly unstable, which is characterized by eigenvalues with a real positive part. In summary, the present work identifies 3 key parameters and studies their effects on determining the thermoacoustic system stability and transient energy growth behaviors. The reported analytical approach can also be applied to a thermoacoustic system with Neumann boundary conditions.

Acknowledgements

The work has been financially supported by Singapore Ministry of Education, under the grant AcR-Tier1-RG91/13-M4011228. We would like to thank Prof. Wolfgang Polifke (TUM, Germany) and Prof. R.I. Sujith (IIT, Madras) for helpful discussion.

Appendix A. Thermoacoustic system with Neumann boundary conditions

The above Galerkin series expansion method can be applied to a thermoacoustic system with Neumann boundary conditions, i.e.

$\partial p / \partial x|_{x=0} = 0$ and $\partial p / \partial x|_{x=L_0} = 0$. Similarly the basis function $\psi(x)$ can be shown as

$$\psi_n(x) = \begin{cases} \cos\left(\frac{\omega_n x}{\bar{c}_1}\right) & 0 \leq x < x_f \\ -\sin\left(\frac{\omega_n(x-L_0)}{\bar{c}_2}\right) \frac{\cos(\omega_n x_f / \bar{c}_1)}{\sin(\omega_n(L_0-x_f)/\bar{c}_2)} & x_f < x \leq L_0 \end{cases} \quad (\text{A.1})$$

The eigenfrequency ω_n can be determined by using the nonlinear relationship as

$$\begin{aligned} \bar{\rho}_2 \bar{c}_2 \sin(\omega_n x_f / \bar{c}_1) \sin\left(\frac{\omega(L_0-x_f)}{\bar{c}_2}\right) \\ = \bar{\rho}_1 \bar{c}_1 \cos\left(\frac{\omega_n x_f}{\bar{c}_1}\right) \cos\left(\frac{\omega(L_0-x_f)}{\bar{c}_2}\right) \end{aligned} \quad (\text{A.2})$$

The system equation can be shown as

$$\frac{d^2 \eta_n}{dt^2} + \zeta_n \frac{d\eta_n}{dt} + \omega_n^2 \eta_n = \omega_n \frac{(\gamma-1)}{\kappa_n} \left[Q_f(t) \cos\left(\frac{\omega_n x_f}{\bar{c}_1}\right) \right] \quad (\text{A.3})$$

where κ_n is given as

$$\kappa_n = \left[\int_0^{L_0} \psi_n^2 dx \right]^{1/2} = \frac{1}{\sqrt{2}} \left[x_f + (L_0 - x_f) \frac{\cos^2\left(\frac{\omega_n x_f}{\bar{c}_1}\right)}{\sin^2\left(\frac{\omega(L_0-x_f)}{\bar{c}_2}\right)} \right]^{1/2} \quad (\text{A.4})$$

By discretizing the flame front into K elements and following the same analysis method introduced, the non-normality and transient energy growth of disturbances in the thermoacoustic system with Neumann boundary conditions can then be investigated. For simplicity, detailed analysis is neglected here.

Appendix B. Coefficient matrices

The coefficient matrices $\mathbf{L}^{TA}$, $\mathbf{L}^{TF}$, $\mathbf{L}^{BA}$ and $\mathbf{L}^{BF}$ are given as

$$\mathbf{L}^{TA} = \begin{bmatrix} 0 & \omega_1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ -\omega_1 & -\zeta_1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_2 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega_2 & -\zeta_2 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & \omega_N \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & -\omega_N & -\zeta_N \end{bmatrix}_{2N \times 2N} \quad (\text{B.1})$$

$$\mathbf{L}^{BA} = \begin{bmatrix} \lambda_1 \frac{\cos(\omega_1 x_f / \bar{c}_1)}{\bar{\rho}_1 \bar{c}_1 \kappa_1 / \Delta \tau} & 0 & \lambda_1 \frac{\cos(\omega_2 x_f / \bar{c}_1)}{\bar{\rho}_1 \bar{c}_1 \kappa_2 / \Delta \tau} & 0 & \dots & \lambda_1 \frac{\cos(\omega_N x_f / \bar{c}_1)}{\bar{\rho}_1 \bar{c}_1 \kappa_N / \Delta \tau} & 0 \\ \lambda_2 \frac{\cos(\omega_1 x_f / \bar{c}_1)}{\bar{\rho}_1 \bar{c}_1 \kappa_1 / \Delta \tau} & 0 & \lambda_2 \frac{\cos(\omega_2 x_f / \bar{c}_1)}{\bar{\rho}_1 \bar{c}_1 \kappa_2 / \Delta \tau} & 0 & \dots & \lambda_2 \frac{\cos(\omega_N x_f / \bar{c}_1)}{\bar{\rho}_1 \bar{c}_1 \kappa_N / \Delta \tau} & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_{K-1} \frac{\cos(\omega_1 x_f / \bar{c}_1)}{\bar{\rho}_1 \bar{c}_1 \kappa_1 / \Delta \tau} & 0 & \lambda_{K-1} \frac{\cos(\omega_2 x_f / \bar{c}_1)}{\bar{\rho}_1 \bar{c}_1 \kappa_2 / \Delta \tau} & 0 & \dots & \lambda_{K-1} \frac{\cos(\omega_N x_f / \bar{c}_1)}{\bar{\rho}_1 \bar{c}_1 \kappa_N / \Delta \tau} & 0 \\ \lambda_K \frac{\cos(\omega_1 x_f / \bar{c}_1)}{\bar{\rho}_1 \bar{c}_1 \kappa_1 / \Delta \tau} & 0 & \lambda_K \frac{\cos(\omega_2 x_f / \bar{c}_1)}{\bar{\rho}_1 \bar{c}_1 \kappa_2 / \Delta \tau} & 0 & \dots & \lambda_K \frac{\cos(\omega_N x_f / \bar{c}_1)}{\bar{\rho}_1 \bar{c}_1 \kappa_N / \Delta \tau} & 0 \end{bmatrix}_{K \times 2N} \quad (\text{B.2})$$

$$\mathbf{L}^{BF} = \begin{bmatrix} -\frac{S_u}{\Delta \tau} & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ S_u \frac{\lambda_2}{\lambda_1} & -\frac{S_u}{\Delta \tau} & 0 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & S_u \frac{\lambda_{K-1}}{\lambda_{K-2}} & -\frac{S_u}{\Delta \tau} & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & S_u \frac{\lambda_K}{\lambda_{K-1}} & -\frac{S_u}{\Delta \tau} \end{bmatrix}_{K \times K} \quad (\text{B.3})$$

$$\mathbf{L}^{TF} = \begin{pmatrix} 0 & 0 & \dots & 0 \\ \chi_{A_f} \frac{\bar{Q}}{\kappa_1} \sin(\frac{\omega_1 X_f}{c_1}) & \chi_{A_f} \frac{\bar{Q}}{\kappa_1} \sin(\frac{\omega_1 X_f}{c_1}) & \dots & \chi_{A_f} \frac{\bar{Q}}{\kappa_1} \sin(\frac{\omega_1 X_f}{c_1}) \\ 0 & 0 & \dots & 0 \\ \chi_{A_f} \frac{\bar{Q}}{\kappa_2} \sin(\frac{\omega_2 X_f}{c_1}) & \chi_{A_f} \frac{\bar{Q}}{\kappa_2} \sin(\frac{\omega_2 X_f}{c_1}) & \dots & \chi_{A_f} \frac{\bar{Q}}{\kappa_2} \sin(\frac{\omega_2 X_f}{c_1}) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \\ \chi_{A_f} \frac{\bar{Q}}{\kappa_N} \sin(\frac{\omega_N X_f}{c_1}) & \chi_{A_f} \frac{\bar{Q}}{\kappa_N} \sin(\frac{\omega_N X_f}{c_1}) & \dots & \chi_{A_f} \frac{\bar{Q}}{\kappa_N} \sin(\frac{\omega_N X_f}{c_1}) \end{pmatrix}_{2N \times K} \quad (\text{B.4})$$

Appendix C. Coefficient matrix Q

Q is a $(2N + K) \times (2N + K)$ diagonal matrix as given as

$$\mathbf{Q} = \begin{pmatrix} \frac{1}{\kappa_1 \sqrt{2/p}} & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\kappa_1 \sqrt{2/p}} & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\kappa_2 \sqrt{2/p}} & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\kappa_2 \sqrt{2/p}} & \dots & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \frac{1}{\kappa_N \sqrt{2/p}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & \frac{1}{\kappa_N \sqrt{2/p}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & \sqrt{\left(\chi_{A_f} \frac{\bar{Q}}{\kappa_1}\right)^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \sqrt{\left(\chi_{A_f} \frac{\bar{Q}}{\kappa_N}\right)^2} \end{pmatrix} \quad (\text{C.1})$$

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